Patterning and Algebra Breakout Sessions

There are three key ideas that we will focus on during the course of the two-day breakout sessions: multiplicative reasoning, generalizing, and multiple representations (prioritizing visual in order to understand and *see* mathematical structure). We will also infuse examples of how to ask questions of students that allows them to formulate conjectures about mathematical relationships and mathematical structure.

We have chosen these three key interconnected concepts because these are the areas that are fundamentally important, but with which students have the most difficult in terms of their algebraic learning.

Multiplicative Thinking:
Research indicates that while most students are able to solve multiplication problems involving small numbers, they tend to rely on additive strategies to solve more complex problems (Siemon & Breed). The transition from additive to multiplicative thinking is neither smooth nor straightforward. When considering the development of algebraic reasoning, multiplicative thinking is fundamental to understanding the idea of "function" as the co-variation between two sets of data – also known as one-to-many correspondence. Many traditional approaches and models for algebraic instruction seem to (inadvertently) support additive thinking, that is, recognition of variation within one set of data (for instance, *for this pattern add three more tiles each time*). Multiplicative thinking, on the other hand, is the recognition of co-variation between two sets of data (for instance, *for any term/position number of the pattern multiply the term by 3 to figure out the number of tiles*).

Generalizing:
Algebra can be thought of as generalizations of laws about relationships between and among numbers and patterns. Part of this understanding includes working with letters as expressions of variables in order to represent *any* case in a relationship – not just a particular missing unknown quantity. A focus is on transitioning from term-to-term generalizations (scalar/recursive or additive thinking) to generalizations that result in explicit statements about mathematical relations between independent and dependent variables. The ability to generalize includes an ability to make near predictions, for instance the 5^{th} term of a pattern, far predictions, for instance the 28^{th} term of a pattern. It also includes the ability to be able to articulate the general rule for a pattern in order to be able to make predictions for *any* term.

Multiple Representations
Observing patterns and relationships lie at the heart of acquiring deep understanding of algebraic reasoning. Past research indicates that working with visual representations (linear growing patterns, diagrams) and deconstructing these in order to identify the relationship between variables have been more successful methods of developing generalized algebraic formulae than either working with number sequences, using ordered tables of values, or memorizing rules for transforming equations. In addition, research suggests that when data is expressed numerically and sequentially (ordered table of values) scalar or term-to-term relationships are likely to be dominant.

When considering the study of algebraic relationships in higher grades, mathematics educators recommend that students be introduced to various representational forms of relationships in order to develop the ability to use these representations effectively as a means of considering quantitative relationships including symbolic notation and graphs. In addition, researchers stress that it is the ability to make *connections* among different representations, specifically symbolic/numeric and graphic ones, that allow students to develop insights for constructing the concept of a mathematical relationship.

For higher grades, we will also be exploring how to support students in constructing an understanding of solving [systems of] equations of the form ax+b=cx+d for which there are one solution, no solution, or infinite solutions. This exploration will be based on students' understanding of linear growing patterns and graphs of linear growing patterns. Research has demonstrated that students who rely only on remembered rules for transforming and solving equations often misapply them, or misinterpret them, or do not think about the meaning of the situations in which they might be successfully applied.[1]

Many of our discussions will revolve around specific practical activities that can be used with all grades so that, as a larger group, we can examine trajectories of learning. Activities include:

- Exploring function machine, robot rules activities
- Building linear growing patterns
- Creating graphs

[1] To solve equations of the form ax+b=cx+d, students are taught a standard procedure using subtraction in order to get the variable terms on the left and the constant on the right, and then dividing by the coefficient of the variable term. However, a more meaningful approach is to compare the different rates of growth as represented by the multipliers, and subtraction to compare the constants, or "where the two lines start on a graph", and finally, divided that number by the rate of growth number. If we think of ax+b and cx+d as two pattern rules for which the value of the multiplier and the constant are different, then the solution to the equation is analogous to finding the position number (x) at which the trend lines of the two rules will intersect. To determine how far apart trend lines "start" on the y-axis, students find the numeric difference between the values of the constant, or (d-b). To find the rate at which they "come together," students find the difference between one multiplier and the other (a-c). To find the position number (x), they divide "how far apart they started" by "the rate at which they come together" or x=(d-b)÷(a-c). This helps to support a more conceptual understanding of *why* the operations of subtraction and division are carried out when solving for x.

- Solving contextual problems

Throughout these activities and tasks we will return consistently to Mason's idea of noticing by listening carefully and fearlessly to one another and responding through our deepening understanding of mathematics.

Proposed supporting articles (so far…):

Ferrini-Mundy, Lappan & Phillips. (2000). Experiences with patterning. In Moses, B (Ed.), *Algebraic Thinking, Grades K-12; Readings from NCTM's School-Based Journals and Other Publications*. Reston, VA: National Council of Teachers of Mathematics, pp. 282-289.

Siemon, D & Breed, M. Assessing multiplicative thinking using rich tasks.

Mason, John (2009). Teaching as disciplined enquiry. *Teachers and Teaching: Theory and practice,* 15(2), pp. 205–223.

Jacobs, L. & Willis, S. The development of multiplicative thinking in young children.

Focused Break Out Session – Patterning and Algebra
Content By Grade Level

The following sessions will be infused with some overarching questions:

1. How can we emphasize situations in which generalizations can be identified and described?

2. When we observe student learning, how can we ask good questions to support conjecturing, testing, proving, theorizing and the construction of generalizations?

3. How can we develop students' natural ability to discern patterns and generalize them?

Wednesday August 17th – 3:15 – 4:45
Introduction to the three key principles (multiplicative thinking, generalizing, and multiple representations) and the kinds of activities we'll use to explore them.

Grades K – 4

Activities: Introduction to multiplicative thinking, generalizing, and multiple representations through the Function Machine and building Linear Growing Patterns.

Multiplicative Thinking
- Children as young as JK can start to look for relationships between data sets
- For example, multiplying a number by 2 might be thought of as doubling the number, adding a number to itself etc. Multiplying a number by 3 might be thought of as doubling the number + number, number + number + number, tripling the number. These relationships can be explored visually and numerically.
- Build on young students' abilities to skip count.

Generalizing
- Find a pattern rule.
- Find a pattern rule that will allow you to predict the out put number/number of tiles from the input number/position number
- Find the input number/position number given the output number/number of tiles if you know the pattern rule
- Near and far predictions (e.g., in a linear growing pattern, the number of tiles at the 10th, 25th, 100th, 1000th position/term)

Multiple Representations
- Connect the function machine and linear growing patterns and understand that both represent the same relationship between quantities

Supporting Questions
- Questions that support students to notice predictable growth

- Questions that support students to articulate relationships. "What patterns do you see? What strategy did you use to find the pattern rule?"
- Questions that support predictions. "How many tiles would there be in the next position? What would it look like? How many tiles might you have at the 125th position? What would it look like? What would the output number be if the input was 20?"
- Questions that extend thinking. "If I wanted to build the 18th position and I had 127 tiles, could I do it? Why or why not? If we had two rules, for instance $3x+5$ and $5x+3$, would the patterns look the same? Why or why not?"
- Questions that encourage students to make connections among representations. "What part of the rule represents the part of the pattern that grows? What part of the rule is responsible for the part of the pattern that stays the same?"

Grades 3 – 6

Activities: Introduction to multiplicative thinking, generalizing, and multiple representations through Robot Rules and building Linear Growing Patterns.

Multiplicative Thinking

- Children can start to look for relationships between data sets
- Building linear growing patterns allows students to develop the concept of a unit (unitizing) – the part of the pattern that is repeated as a function of the position number
- The Robot Game and Pattern Building support students to go from additive reasoning (looking at change from one iteration of the pattern to the next – e.g., "add 3 more tiles each time") to multiplicative reasoning (the number of tiles is equal to the position number times 3 – emphasizing *one-to-many correspondence*)

Generalizing

- Find a pattern rule
- Find a pattern rule that will allow you to predict the out put number/number of tiles from the input number/position number
- Find the input number/position number given the output number/number of tiles if you know the pattern rule
- Near and far predictions (e.g., in a linear growing pattern, the number of tiles at the 10th, 25th, 100th, 1000th position/term).

Multiple Representations

- Make connections among the robot chart, linear growing patterns, and pattern rules and understand that all represent the same relationship between quantities

Supporting Questions

- Questions that support students to notice predictable growth.
- Questions that support students to articulate relationships. "What patterns do you see?" "What strategy did you use to find the pattern rule?"

- Questions that support predictions. "How many tiles would there be in the next position? What would it look like? How many tiles might you have at the 125th position? What would it look like? What would the output be if the input number was 23?"
- Questions that extend thinking. "If I wanted to build the 18th position and I had 127 tiles, could I do it? Why or why not?" "If we had two rules, for instance $3x+5$ and $5x+3$, would the patterns look the same? Why or why not?"
- Questions that encourage students to make connections among representations. "What part of the rule represents the part of the pattern that grows? What part of the rule is responsible for the part of the pattern that stays the same?"

Grade 5 – 8

Activities: Introduction to multiplicative thinking, generalizing, and multiple representations through Robot Rules and building Linear Growing Patterns.

Multiplicative Thinking

- Focus on relationships between data sets
- Building linear growing patterns allows students to develop the concept of a unit (unitizing) – the part of the pattern that is repeated as a function of the position number
- The Robot Game and Pattern Building support students to go from additive reasoning (looking at change from one iteration of the pattern to the next – e.g., "add 3 more tiles each time") to multiplicative reasoning (the number of tiles is equal to the position number times 3 – this is one-to-many correspondence)
- Graphs are another way of capturing this relationship

Generalizing

- Build patterns – Robot Rules game – identify and express a generalization
- Find a pattern rule that will allow you to predict the out put number/number of tiles from the input number/position number
- Find the input number/position number given the output number/number of tiles if you know the pattern rule
- Make near and far predictions (e.g., in a linear growing pattern, the number of tiles at the 10th, 25th, 100th, 1000th position/term).
- Begin to understand the relationship between independent variables (input numbers/position numbers) and dependent variables (output numbers/number of tiles)
- Near and far predictions (e.g., in a linear growing pattern, the number of tiles at the 10th, 25th, 100th, 1000th position/term).

Multiple Representations

- Make connections among the robot chart, linear growing patterns, and pattern rules
- Make connections to graphical representations (multiplier →slope, constant → y-intercept)

- Understand that all representations depict the same relationship between quantities – different representations highlight different aspects of the relationship (what aspects of the relationship are more easily seen through a graphical representation as opposed to a pattern rule/numeric expression?)

Supporting Questions

- Questions that support students to notice predictable growth.
- Questions that support students to articulate relationships. "What patterns do you see? What strategy did you use to find the pattern rule?"
- Questions that support predictions. "How many tiles would there be in the next position? What would it look like? How many tiles might you have at the 125th position? What would it look like? What would the output be if the input number was 23?"
- Questions that extend thinking. "If I wanted to build the 18th position and I had 127 tiles, could I do it? Why or why not?" "If we had two rules, for instance x3+5 and x5+3, would the patterns look the same? Why or why not?"
- Questions that encourage students to make connections among representations. "What part of the rule represents the part of the pattern that grows? What part of the rule is responsible for the part of the pattern that stays the same?"

Grades 7 – 10

Activities: Introduction to multiplicative thinking, generalizing, and multiple representations through Robot Rules and building Linear Growing Patterns.

Multiplicative Thinking

- Focus on relationships between data sets
- Building linear growing patterns allows students to develop the concept of a unit (unitizing) – the part of the pattern that is repeated as a function of the position number
- The Robot Game and Pattern Building support students to go from additive reasoning (looking at change from one iteration of the pattern to the next – e.g., "add 3 more tiles each time") to multiplicative reasoning (the number of tiles is equal to the position number times 3 – this is one-to-many correspondence)
- Graphs are another way of capturing this relationship

Generalizing

- Build patterns – Guess My Rule game
- Find a pattern rule that will allow you to predict the out put number/number of tiles from the input number/position number
- Find the input number/position number given the output number/number of tiles if you know the pattern rule
- Begin to understand the relationship between independent variables (input numbers/position numbers) and dependent variables (output numbers/number of tiles)

- Make near and far predictions (e.g., in a linear growing pattern, the number of tiles at the 10^{th}, 25^{th}, 100^{th}, 1000^{th} position/term).

Multiple Representations

- Make connections among the robot chart, linear growing patterns, and pattern rules
- Understand that all representations depict the same relationship between quantities – different representations highlight different aspects of the relationship (what aspects of the relationship are more easily seen through a graphical representation as opposed to a pattern rule/numeric expression?)

Supporting Questions

- Questions that support students to notice predictable growth.
- Questions that support students to articulate relationships. “What patterns do you see? What strategy did you use to find the pattern rule?”
- Questions that support predictions. “How many tiles would there be in the next position? What would it look like? How many tiles might you have at the 125^{th} position? What would it look like? What would the output be if the input number was 23?”
- Questions that extend thinking. “If I wanted to build the 18^{th} position and I had 127 tiles, could I do it? Why or why not?” “If we had two rules, for instance x3+5 and x5+3, would the patterns look the same? Why or why not?”
- Questions that encourage students to make connections among representations. “What part of the rule represents the part of the pattern that grows? What part of the rule is responsible for the part of the pattern that stays the same?”

Grades 9 – 12

Activities: Introduction to multiplicative thinking, generalizing, and multiple representations through Robot Rules and building Linear Growing Patterns. Given three rules with the same multiplier (coefficient), or three rules with the same constant, predict what the trend lines of will look like.

Multiplicative Thinking

- Focus on relationships between data sets
- Building linear growing patterns allows students to develop the concept of a unit (unitizing) – the part of the pattern that is repeated as a function of the position number
- The Robot Game (unordered table of values) and Pattern Building activities support students to solidify an understanding of multiplicative relationships as the one-to-many correspondence between one set of data and another
- Introduce graphs as another way of capturing this relationship

Generalizing

- Find and express a rule for a pattern using an equation, where y is the number of tiles, and x is the position number of the pattern

- Near and far predictions (e.g., in a linear growing pattern, the number of tiles at the 10^{th}, 25^{th}, 100^{th}, 1000^{th} position/term).
- Develop an understanding of *x* as a generalized number that can take a range of values (bridge from the idea of unknown to that of variable) – so *x* is a*ny* position number of a pattern, and *y* is the number of tiles dependent on that number, given the pattern rule
- Develop understanding of the relationship between independent variables (input numbers/position numbers) and dependent variables (output numbers/number of tiles)
- Construct generalizations about the role of the multiplier, the role of the constant and how these affect the trend line of the graph (multiplier →slope, constant → y-intercept)
- Compare relationships between pattern rules/functions.

Multiple Representations

- Understand that all representations depict the same relationship between quantities – different representations highlight different aspects of the relationship (what aspects of the relationship are more easily seen through a graphical representation as opposed to a pattern rule/numeric expression?)
- Given a pattern rule/equation making predictions about the trend line on the graph

Supporting Questions

- Questions that support students to notice predictable growth.
- Questions that support students to articulate relationships. “What patterns do you see? What strategy did you use to find the pattern rule?”
- Questions that support predictions. “How many tiles would there be in the next position? What would it look like? How many squares might you have at the 125^{th} position? What would it look like?”
- Questions that extend thinking. “If I wanted to build the 18^{th} position and I had 127 tiles, could I do it? Why or why not?” “If we had two rules, for instance x3+5 and x5+3, would the patterns look the same? Why or why not?”
- Questions that encourage students to make connections among representations. “What part of the rule represents the part of the pattern that grows? What part of the rule is responsible for the part of the pattern that stays the same? What part of the rule is responsible for the steepness of the trend line? What part of the rule is responsible for where the line “starts” on the graph?”
- Questions that encourage students to notice similarities and differences. “When you compare three pattern rules, the patterns and the graphs what is the same about all three representations and what is different?”

INCLUDE 3D REPRESENTATIONS – for instance, given a coefficient, can student visualize a point or trend line in 3D space? Using cardboard boxes and straws to represent slope – then check using autograph.

Thursday August 18 – Breakout Session 1 – 10:45 – 12:00 [*We may want to move this session to a later time, after groups have had a chance to work on more of the activities*]

Descriptive feedback.
Learning to use our Algebra Eyes and Ears – participants in each break out session will view videotapes of student learning and view student work to determine evidence of either multiplicative thinking, generalizing, or use of multiple representations. Participants from each of the three structured viewing groups will then meet to discuss what they saw, what this tells them about the students' level of understanding, and what they would do next.

Thursday August 18 – Breakout Session 2 – 3:15 – 4:45
Pattern Building and Graphing

Grades K – 4

Activities: Building Linear Growing Patterns and creating graphs. Formulate rules for pattern *when only one position of the pattern is given*. Create graphical representations of patterns

Multiplicative thinking

- Continue to explore the concept of "unitizing" with patterns of one colour
- Introduce graphing and story contexts for further exploring one-to-many correspondence

Generalizing

- Formulate conjectures about pattern rules when only one position is shown
- Explore the relationship between similar patterns and different pattern rules, and similar rules and different patterns
- Find the input number/position number given the output number/number of tiles if you know the pattern rule
- Identify the generalized rule in a contextual problem and use it to find particular missing values

Multiple Representations

- Compare two colour and one colour patterns
- Make connections between the multiplier and the constant in the rule and in the patterns
- Make connections among the pattern rule, patterns, and graphs
- Make connections among other representations and story contexts

Supporting Questions

- Questions that support conjecturing. "What is a rule that this pattern could be following? How would you build other positions of this pattern? Are there other rules this pattern might be following?"
- Questions to connect the pattern rule, pattern, and graph. "Are the points on the graph in a straight line? Why? Why do different rules make trend lines that go up more

steeply? What rule would make a steep slope? What rule could make the flattest line?"
- Questions that support making connections to story contexts. "In the story, what was the part that was growing? What part stayed the same? How is this shown on the graph?"

Grades 3 – 6

Activities: Building Linear Growing Patterns and creating graphs, formulate rules for pattern when only one position of the pattern is given. Create graphical representations of patterns. Given three rules with the same multiplier (coefficient), or three rules with the same constant, predict what the trend line of the graphs will look like.

Multiplicative Thinking

- Continue an exploration of developing multiplicative thinking through exploring patterns using only one colour of tile
- Exploring one-to-many correspondence in different "story" contexts

Generalizing

- Formulate conjectures about pattern rules when only one position is shown
- Finding a pattern rule that will allow you to predict the out put number/number of tiles from the input number/position number
- Find the input number/position number given the output number/number of tiles if you know the pattern rule
- Construct a generalized understanding about the role of the multiplier, the role of the constant and how these affect the trend line of the graph
- Identifying the generalized rule in a contextual problem and use that to find particular missing values

Multiple Relationships

- One colour tile patterns allow for the merging of numeric and visual understanding of linear relationships
- Make connections among pattern rules, patterns and graphs
- Given a pattern rule/equation making predictions about the trend line on the graph
- Construct generalizations about the role of the multiplier, the role of the constant and how these affect the trend line of the graph (multiplier →slope, constant → y-intercept)
- Compare relationships between pattern rules/functions.
- Introduce story problems as another way of representing linear relationships
- Understand that all representations depict the same relationship between quantities – different representations highlight different aspects of the relationship (what aspects of the relationship are more easily seen through a graphical representation as opposed to a pattern rule/numeric expression?)

Supporting Questions

- Questions that support conjecturing. "What is a rule that this pattern could be following? How would you build other positions of this pattern? Are there other rules this pattern might be following?"
- Questions to connect the pattern rule, pattern, and graph. "Are the points on the graph in a straight line? Why? Why do different rules make trend lines that go up more steeply? What rule would make a steep slope? What rule could make the flattest line?"
- Questions that support making connections to story contexts. "In the story, what was the part that was growing? What part stayed the same? How is this shown on the graph?

Grade 5 – 8

Activities: Create graphs of linear growing patterns. Given three rules with the same multiplier (coefficient), or three rules with the same constant, predict what the trend lines will look like.

Multiplicative Thinking

- Continue to focus on multiplicative relationships between data sets
- Explore graphs as another way of capturing this relationship

Generalizing

- Formulate conjectures about pattern rules when only one position is shown (?)
- Construct a generalized understanding about the role of the multiplier, the role of the constant and how these affect the trend line of the graph

Multiple Representations

- Make connections among pattern rules, patterns and graphs
- Given a pattern rule/equation make predictions about the trend line on the graph
- Construct generalizations about the role of the multiplier, the role of the constant and how these affect the trend line of the graph (multiplier →slope, constant → y-intercept)
- Compare relationships between pattern rules/functions.
- Introduce story problems as another way of representing linear relationships
- Understand all representations depict the same relationship between quantities – different representations highlight different aspects of the relationship (what aspects of the relationship are more easily seen through a graphical representation as opposed to a pattern rule/numeric expression?)

Supporting Questions

- Questions to foster making predictions about near, far and any case.
- Questions that encourage students to make connections among representations. "What part of the rule represents the part of the pattern that grows? What part of the rule is responsible for the part of the pattern that stays the same? What part of the rule is

responsible for the steepness of the trend line? What part of the rule is responsible for where the line "starts" on the graph?"
- Questions that encourage students to notice similarities and differences. "When you compare three pattern rules, the patterns and the graphs what is the same about all three representations and what is different?"

Grades 7 – 10

Activities: Create graphs of linear growing patterns. Given three rules with the same multiplier (coefficient), or three rules with the same constant, predict what the trend line of the graphs will look like.

Multiplicative Thinking
- Continue to focus on relationships between data sets
- Explore graphs are another way of capturing this relationship

Generalizing
- Construct generalized understanding about the role of the multiplier, the role of the constant and how these affect the trend line of the graph (multiplier →slope, constant → y-intercept)
- Develop an understanding of x as a generalized number that can take a range of values (bridge from the idea of unknown to that of variable) – so, for example, x is *any* position number of a pattern, and y is the number of tiles dependent on that number, given the pattern rule

Multiple Representations
- Make connections among pattern rules, patterns and graphs
- Given a pattern rule/equation make predictions about the trend line on the graph
- Construct generalizations about the role of the multiplier, the role of the constant and how these affect the trend line of the graph (multiplier →slope, constant → y-intercept)
- Compare relationships between pattern rules/functions.
- Understand all representations depict the same relationship between quantities – different representations highlight different aspects of the relationship (what aspects of the relationship are more easily seen through a graphical representation as opposed to a pattern rule/numeric expression?)

Supporting Questions
- Questions to foster making predictions about near, far and any case.
- Questions that encourage students to make connections among representations. "What part of the rule represents the part of the pattern that grows? What part of the rule is responsible for the part of the pattern that stays the same? What part of the rule is responsible for the steepness of the trend line? What part of the rule is responsible for where the line "starts" on the graph?"

- Questions that encourage students to notice similarities and differences. "When you compare three pattern rules, the patterns and the graphs what is the same about all three representations and what is different?"

Grades 9 – 12

Activities: Construct graphs of pattern rules for which the multiplier and constant are different. Predict what the graph will look like.

Multiplicative Thinking

- Continue to focus on relationships between data sets
- Compare pattern rules (algebraic equations)

Generalizing

- Develop an understanding of *x* as a generalized number that can take a range of values (bridge from the idea of unknown to that of variable) – so for example *x* is *any* position number of a pattern, and *y* is the number of tiles dependent on that number, given the pattern rule
- Predict the point of intersection when comparing two pattern rules/equations

Multiple Representations

- Continue to explore connections among representations.
- Move to solving equations based on an understanding of what x and y actually represent – having students <u>construct</u> an understanding for solving equations of the form ax+b=cx+d. – this is in contrast to having students memorize rules for transforming and solving equations

Supporting Questions

- Questions that support making connections among representations. "What does it mean when trend lines intersect on a graph?"
- Questions that support making predictions. "Given two rules, is it possible to predict where the point of intersection will be on the graph?"
- Questions that support generalizations. "What needs to be true in order for two rules to have trend lines that intersect?"

Thursday August 18 – Breakout Session 3 – 7:00 – 8:30
Introducing the Swimming Pool Problem.

Grades K – 4

Activities K-2: Story problems, patterns in context, building patterns, graphs.

Multiplicative Thinking
- Explore relationships between quantities in different contexts

Generalization
- Find pattern rules
- Make near and far predictions

Multiple representations
- Make connections among story contexts, graphs, patterns and pattern rules
- Explore different kinds of relationships (not necessarily linear)

Supporting Questions
- Questions that support conjecturing. "What is a rule that this pattern could be following? How would you build other positions of this pattern? Are there other rules this pattern might be following?"
- Questions to connect the pattern rule, pattern, and graph.
- Questions that support making connections to story contexts. "In the story, what was the part that was growing? What part stayed the same? How is this shown on the graph?"

Activities 3-4: Exploring the Swimming Pool Problem with models and drawings.

Multiplicative Thinking
- Find visual and numeric patterns in the relationship among the blue tiles, the white tiles, and between the blue and the white tiles

Generalization
- Find and articulate a pattern rule (rules)
- Foster the ability to make conjectures about the relationships between and among the two colours of tiles

Multiple representations
- Patterns and pattern rules and tables (as a way of recording data)
- Compare two kinds of relationships (linear and quadratic)

Supporting Questions
- Questions that support making generalizations. "What might be a rule for the white tiles? What might be a rule for the blue tiles?"

- Questions that encourage making predictions about near, far, and any pool (white or blue tiles).
- Questions that encourage predictions based on observed relationships between the two patterns. "If there are 32 white tiles in the border, how many blue tiles would there be? If there are 36 blue tiles, how many white tiles would there be? Can you make a square with 49 blue tiles?"

Grades 3 – 6

Activities: Exploring the Swimming Pool Problem with models, drawings, and graphs.

Multiplicative Thinking

- Counting tiles – transitioning from additive to multiplicative thinking. For instance when counting tiles for the 2^{nd}, 3^{rd}, 4^{th}, nth pool beginning to understand the structure of the array, adding rows or columns.
- White tiles – "adding 4 each time" vs. "multiplying the side by 4 plus 4"

Generalization

- Identifying and articulating the pattern rule (rules)
- Make predictions based on pattern rules

Multiple Representations

- Compare two kinds of relationships (linear and quadratic)
- Explore equivalence of expression – multiple rules for the Swimming Pool Problem.

Supporting Questions

- Questions that support making generalizations. "What might be a rule for the white tiles? What might be a rule for the blue tiles?"
- Questions that encourage making predictions about near, far, and any pool (white or blue tiles).
- Questions that encourage predictions based on observed relationships between the two patterns. "If there are 32 white tiles in the border, how many blue tiles would there be? If there are 36 blue tiles, how many white tiles would there be? Can you make a square with 49 blue tiles?"
- Questions that connect the different representations. "What do you think a graph of the white tiles would look like? What do you think a graph of the blue tiles would look like? Will the trend lines be similar or different? Will the trend lines intersect? Why?"

Grade 5 – 8

Activities: Exploring the Swimming Pool Problem with models, drawings, and graphs.

Multiplicative Thinking

- (Grade 5 and 6) Transitioning from additive to multiplicative thinking. For instance when counting tiles for the 2nd, 3rd, 4th, nth pool beginning to understand the structure of the array, adding rows or columns.
- Find rules for the white tiles – "adding 4 each time" vs. "multiplying the side by 4 plus 4"

Generalizations

- Identify and articulate the pattern rule (rules)
- Make predictions based on pattern rules

Multiple Representations

- Compare two types of relationships – the quadratic relationship of the square numbers of the blue tiles, and the linear relationship of the white tiles
- Exploring equivalence of expression – multiple rules for the Swimming Pool Problem (the white tiles)
- Explore the graphical representation for the two rules

Supporting Questions

- Questions that support making generalizations. "What might be a rule for the white tiles? What might be a rule for the blue tiles?"
- Questions that encourage making predictions about near, far, and any pool (white or blue tiles).
- Questions that encourage predictions based on observed relationships between the two patterns. "If there are 32 white tiles in the border, how many blue tiles would there be? If there are 36 blue tiles, how many white tiles would there be? Can you make a square with 49 blue tiles?"
- Questions that connect the different representations. "What do you think a graph of the white tiles would look like? What do you think a graph of the blue tiles would look like? Will the trend lines be similar or different? Will the trend lines intersect? Why?

Grades 7 – 10

Activities: Exploring the Swimming Pool Problem with models, drawings, and graphs.

Multiplicative Thinking

- Discerning different kinds of relationships
- White tiles – "adding 4 each time" vs. "multiplying the side by 4 plus 4"
- Comparing two types of relationships – the quadratic relationship of the square numbers of the blue tiles, and the linear relationship of the white tiles

Generalizations

- Identify and articulate the pattern rule (rules)
- Make predictions based on pattern rules

Multiple Representations

- Explore equivalence of expression – multiple rules for the Swimming Pool Problem (the white tiles)
- Explore the graphical representation for the two rules
- Determine the meaning of the point of intersection (ability to predict the point of intersection?)

Supporting Questions

- Questions that support making generalizations. “What might be a rule for the white tiles? What might be a rule for the blue tiles?”
- Questions to extend thinking to explore equivalence. “How many different rules can you find for the white tiles? The blue tiles?”
- Questions that encourage making predictions about near, far, and any pool (white or blue tiles).
- Questions that encourage predictions based on observed relationships between the two patterns. “If there are 32 white tiles in the border, how many blue tiles would there be? If there are 36 blue tiles, how many white tiles would there be? Can you make a square with 49 blue tiles?”
- Questions that connect the different representations. “What do you think a graph of the white tiles would look like? What do you think a graph of the blue tiles would look like? Will the trend lines be similar or different? Will the trend lines intersect? Why?”

Grades 9 – 12

Activities: Exploring the Swimming Pool Problem with models, drawings, and graphs.

Multiplicative Thinking

- Compare two types of relationships – the quadratic relationship of the square numbers of the blue tiles, and the linear relationship of the white tiles

Generalizations

- Identify and articulate the pattern rule (rules)
- Make predictions based on pattern rules

Multiple Representations

- Explore equivalence of expression – multiple rules for the Swimming Pool Problem (the white tiles)
- Explore the graphical representation for the two rules
- Determine the meaning of the point of intersection (ability to predict the point of intersection?)

Supporting Questions

- Questions that support making generalizations. "What might be a rule for the white tiles? What might be a rule for the blue tiles?"
- Questions to extend thinking to explore equivalence. "How many different rules can you find for the white tiles? The blue tiles?"
- Questions that encourage making predictions about near, far, and any pool (white or blue tiles).
- Questions that encourage predictions based on observed relationships between the two patterns. "If there are 32 white tiles in the border, how many blue tiles would there be? If there are 36 blue tiles, how many white tiles would there be? Can you make a square with 49 blue tiles?"
- Questions that connect the different representations. "Where is the multiplier of your rule represented in the tiles? Where is the constant part of your rule represented in the tiles? What do you think a graph of the white tiles would look like? What do you think a graph of the blue tiles would look like? Will the trend lines be similar or different? Will the trend lines intersect? Why?"